The method of Ref. 2 provides a much better agreement with the CT control than supposed by Ref. 1. The method of Ref. 5 appears precise enough and comparable to the one proposed in Ref. 1. For this reason, and for conciseness, the MPWM strategy of Ref. 6 has not been considered.

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Reply by the Authors to Franco Bernelli-Zazzera and Paolo Mantegazza

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As the comment states, cutting off a pulse at the sampling instant $t + \Delta$ when the pulse width is greater than $\Delta - \tau$ is not consistent with the principle of equivalent area (PEA). We overlooked this point and did not find the overlap method employed in the simulation in Ref. 1; however, it is not explicitly mentioned in Ref. 1. Although the overlapping is consistent with the PEA, it is not a theoretical result. In fact, the optimal delay time is derived from the state-error analysis at the sampling instant $t + \Delta$, where the pulse width is assumed to be smaller than $\Delta - \tau$, that is, no overlapping is assumed. Therefore, there is no guarantee that the method always works well, thought it may give better results than cutting off the pulse. As pointed out in Ref. 2, the method of Ref. 1 "does not work well for the systems that operate with smaller control signals, which require longer pulse durations." In such a case, the firing time often overlaps with the next sampling interval. The reason for the poor performance is that in Ref. 1 $\Psi(-\delta)$ is approximated by $\boldsymbol{I}$, whereas the method of Ref. 2 and ours equivalently approximate it by $\boldsymbol{I} - \boldsymbol{A}\delta/2$; thereby, the better delay time is obtained as well as the pulse width that is strictly consistent with the PEA. The authors of Ref. 1 later improved their method in Ref. 3. As a matter of fact, the delay times provided by their new method in the one-pulse case and by the methods of Refs. 2 and 4 are exactly the same. Reference 2 first presented the delay time. Unfortunately we did not know the paper. However, now that the better delay time is found, the delay time of Ref. 1, hence the overlap method, will not be used. However, we admit that the overlap method works much better than the pulse cutoff.

We reply to the other points of the comment as follows.

1) As to the steady-state error in Fig. 4a of Ref. 4, because the computed pulse width is actually greater than $\Delta - \tau$ and we cut off the pulse at the next sampling instant $t + \Delta$ or more precisely $t + \Delta$ plus computational delay, the PEA does not hold for the pulse. This means that the control power is not enough to reproduce the state responses of the [pulse-amplitude modulation] PAM control system. For this reason, the steady-state error remains. The authors of Ref. 1 state in the comment that "the [pulse-width modulation] PWM steady-state response is on average equal to that of [discrete time] DT control, regardless of the pulse delay. . . . " However, this statement may lead to a misunderstanding because on average the PWM control does not always achieve no steady-state error. In fact, in the case of no input delay in Fig. 3 (Ref. 4), where neither firing time saturation nor overlap occurs, the steady state eror exists. The steady-state analysis shown by Eqs. (18–27) indicates that if the state responses of the PAM control system agree with those of the PWM control system at every sampling time in the steady-state, then the preceding statement holds. However, the analysis does not show that the PWM control brings the system to the same steady states as those that the PAM brings it to. The steady-state output error may disappear, if the PAM controller or the plant has integral action. This is the case with the example of Ref. 4; however, zero steady-state error is not achieved due to the pulse cutoff in Fig. 4a.

2) As to computational delay, because the method of Ref. 1 gives a constant delay time, the effect of computational delay can be alleviated by reducing the firing delay time by the computational delay time. Also in the method of Ref. 4, however, when the delay time given by Eq. (21), which is variable, is larger than the computational delay, the same logic is applicable. Even when it is smaller than the computational delay, the effect of the delay can be minimized by firing a pulse immediately after the computational delay.

In any case, we admit that cutting off a pulse results in the worse evaluation than should be by taking the overlap method. We also regret not having referred the reader to Refs. 2 and 3 that derived the same delay time as Ref. 1. We appreciate their comment.

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